Response of blood flow through an atherosclerotic artery in the presence of magnetic field using Bingham plastic fluid

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Abstract: The present numerical model was developed to determine the effect of magnetic field on blood flow through an axially non-symmetric but radially symmetric atherosclerotic artery. Blood rheology was described by a weak form Bingham plastic equation. The effect of magnetic field is considered in the transverse direction of blood flow and viscosity of blood is taken as radial co-ordinate dependent. It was disclosed that in the presence of magnetic field, blood did not drastically change the flow patterns, but caused an appreciable decrease in the shear stresses and a slightly lower resistance to flow.

Key words: Resistance to flow, Magnetic field, Wall shear stress, Bingham plastic fluid, Atherosclerotic artery.

Introduction:

Many cardiovascular diseases particularly atherosclerosis, which are responsible for the death of people, are closely related to the nature of the blood movement and the dynamic behavior of blood vessel. From medical survey it is well known fact that more than eighty five percent of the total deaths of people are due to the diseases of blood vessel walls. Atherosclerosis is the leading cause of morbidity and mortality. It is a progressive disease characterized by localized plaques that form within the artery wall. As the disease progresses, these plaques enlarge and either directly or indirectly, lead to impairment of blood flow. This in turn can have serious consequences, such as blockage of the coronary arteries and carotid arteries. Atherosclerotic lesions are found preferentially at specific sites in the arterial system, typically near bends, bifurcations, and other reasons characterized by complex blood flow patterns. These observations, as well as data about the response of endothelial cells to mechanical forces, suggest that the hemodynamic, i.e. fluid dynamic related phenomena play a role in the initiation and progression of the atherosclerosis. It is well known that the external magnetic field has considerable effect on the biological system of human. The magnetic behaviour of blood is justified due to the haemoglobin molecule, a form of iron oxides, which is present at a uniquely high concentration in the mature red blood cells [1]. It is found that the blood possess the property of diamagnetic material when oxygenated and paramagnetic when deoxygenated. In an arterial constriction blood viscosity increases due to the conservation of mass, producing increased wall shear stress in the region of the blood acceleration. Several attempts have been made in the literature to study the effect of stenosis on the blood flow characteristics, including the important contribution of Young [2], Nerem [3], Haldar [4], Bitoun and Bellet [5], Chakravarty [6], Srivastava and Saxena [7], Srivastava [8] have studied the effect of stenosis on the resistance to flow through artery by considering the behaviour of blood as a Casson’s fluid model. The response of blood flow through an artery under stenotic conditions has been attempted by Srivastava [9] and Shukla et al., [10]. But the problem of flow under magnetic effects becomes much more difficult through artery with stenosis at some region and very few researchers have attempted the flow problem under magnetic effects.

A little attention have been made by Tandon and Pal [11], Sud et al., [12] and Mazumdar et al., [13] to study the effect of magnetic field on physiological fluid flows. It has been found that with the help of magnets, the flow of blood in arteries is properly regulated with the regulation in flow. It has also been reported by Barnothy [14] that the biological systems are affected by magnetic field. Sud and Sekhon [15] and Tzitzilakis [16] have analysed the effect of magnetic on blood flow through the human arterial system. Amos and Ogulu [17] concluded a similar study in which they obtained numerical results for the stream function and vorticity using the Gelarkin technique of the finite element method. The effect of an externally applied magnetic field over the flow characteristics of blood in a single stenosed artery has been analysed by Haldar [18]. Computational modelling of flow in diseased arteries using realistic geometries derived from magnetic resonance imaging is gaining favour as a tool for understanding and predicting cardiovascular disease Secomb [19] and Vittorio et al., [20]. This is because in vivo measurements of the flow field in an artery can be costly and are only possible for arteries that are easily accessible. The published literature on the stenosis further reveals that
very few studies are concerned with the problem of symmetric stenosis. In an actual situation, however, the increase in the arterial wall thickness would not be symmetrical. To generalize the problem further, an attempt is therefore made in the present investigation. In this study a mathematical model is proposed to describe blood flow through an axially non-symmetric but radially symmetric stenosed artery when blood is represented as Bingham plastic fluid and a uniform magnetic field is applied on the flow.

**Formulation of the problem:**

We have considered an artery having mild stenosis. The flow of blood is assumed to be steady, laminar and fully-developed. Blood is taken as a Bingham plastic fluid. It is assumed that stenosis is symmetrical about the axis but non-symmetrical with respect to radial co-ordinates. The mathematical expression for geometry can be written as,

\[ R'(z) = R_0 \left[ 1 - A \left[ (L_0 - d') - (z' - d')^m \right] \right], \quad d' \leq z' \leq d' + L_0 \]

\[ = R_0, \quad \text{otherwise}, \]

where

\[ A = \frac{\delta_s m^m}{R_0 (L_0)^m (m-1)} \]

- \( R_0 \): Radius of normal tube
- \( R'(z) \): Radius of stenotic region
- \( L' \): The length of the artery
- \( L_0' \): The length of the stenosis
- \( d' \): Distance between equispaced points
- \( \delta_s \): Maximum height of stenosis
- \( m \): Parameter determining the shape of stenosis (\( m \geq 2 \))

The schematic diagram of the flow is given by fig. (1)

![Fig (1) Geometry of Stenosis](image)

**Bingham plastic fluid model**

For Bingham plastic fluid, the stress-strain relation is given by

\[ \tau' = \tau'_0 + \mu' \left( \frac{du'}{dr} \right) \]

where

\[ \tau' = \left( -\frac{dp'}{dz'} \frac{r'}{2} \right), \quad \tau'_0 = \left( -\frac{dp' R_p'}{dz'} \frac{r'}{2} \right) \]

- \( u' \): axial velocity
- \( \mu' \): viscosity of fluid
- \( -\frac{dp'}{dz'} \): pressure gradient

Following boundary conditions are introduced to solve the above equations,
\[
\left( \frac{\partial u'}{\partial r} \right) = 0 \quad \text{at} \quad r' = 0, \quad u' = 0 \quad \text{at} \quad r' = R(z)
\]

\( \tau' \) is finite \( \text{at} \quad r' = 0 \) \hspace{1cm} (3)

\( P' = P_0 \quad \text{at} \quad z' = 0, \quad P' = P_L \quad \text{at} \quad z' = L \)

**Governing equations:**

Governing equation can be written as:

\[
\left( \frac{\partial P}{\partial Z} \right) + \frac{1}{r} \frac{\partial}{\partial r}\left( \mu' r \frac{\partial u'}{\partial r} \right) + \left( J \times B \right) = 0
\] \hspace{1cm} (4)

\( J = \sigma (E' + u' \times B) \) and \( \mu' = \mu_0 \left( \frac{r}{R_0} \right)^{(-M)} \)

**Non dimensional scheme:**

\[
R = \left( \frac{R'}{R_0} \right), \quad \mu = \left( \frac{\mu'}{\mu_0} \right), \quad r = \left( \frac{r'}{R_0} \right), \quad L_0 = \left( \frac{L'}{L} \right), \quad \sigma = \left( \frac{\sigma}{R_0} \right),
\]

\[
Re = \left( \frac{\rho U_0 R_0}{\mu_0} \right), \quad d = \left( \frac{d'}{L} \right), \quad z = \left( \frac{z'}{L} \right), \quad P = \left( \frac{P}{\rho U_0^2} \right), \quad u = \left( \frac{u'}{U_0} \right)
\] \hspace{1cm} (6)

The governing equations and boundary conditions are transformed to:

\[
R(z) = 1 - A[L_0^{m-1}(z - d) - (z - d)^m], \quad \text{d} \leq z \leq d + L_0
\]

\[
= 1, \quad \text{otherwise,}
\] \hspace{1cm} (7)

Where, \( A = \frac{\delta}{R_0 L_0^{m(m-1)}} \)

\[
r^M \left( \frac{\partial^2 u}{\partial r^2} \right) + (1 - M)r^{-(1+M)} \left( \frac{\partial u}{\partial r} \right) - H^2 u = Re \left( \frac{\partial p}{\partial z} \right)
\] \hspace{1cm} (8)

Where, \( J = \sigma (E + u \times B) \), \( \mu = r^{-M} \) and \( H^2 = \left( \frac{B_0 R_0^2 \sigma}{\mu_0} \right) \)

\[
\tau = \tau_0 + \mu \left( \frac{du}{dr} \right)
\]

where \( \tau = \left( \frac{dp}{dz} \right), \quad \tau_0 = \left( \frac{dp}{dz} \right) \)

\[
\frac{R}{2}
\] \hspace{1cm} (9)
\[
\frac{\partial u}{\partial r} = 0 \quad \text{at} \quad r = 0
\]
\[
u = 0 \quad \text{at} \quad r = R(z)
\]

Solution of the problem

Solving for \(u\) from equation (8) and (9) and using boundary conditions (10), obtains,
\[
R_c \leq r \leq R
\]
\[
u = \left(\frac{R_c}{4 \mu} \frac{dP}{dz}\right)((R^2 - r^2) - \frac{8 (R_c^{1/2} - R_c^{3/2})((r - R_c^{1/2})^2 + H^2)((r - R_c^{3/2})^4 + H^2)(R - R_c)^2}{3 (1 + R_c^2)^2}
\]
\[
+ \frac{(4 R_c^4 + H^2)(r - R_c)(R - R_c)^2}{(1 + R_c^2)} r^4
+ \frac{(5 R_c + H^2)}{(1 + R_c^2)} r^{3/2} (R - R_c)^2
\]
\[
+ \frac{(r - R_c)(8 R_c + H^2)(R - R_c)^2}{(1 + R_c^2)} r^4
\]
\[
(15(r - R_c) + (r - R_c)^3 H^2)(8(r - R_c) + H^2)(R - R_c)^2) \right)^{3/2}
\]
\[
+ \frac{(R - R_c)^2}{(1 + R_c^2)} \right)^{1/2}
\]
\[
+ \frac{(5 R_c + H^2)(r - R_c)(R - R_c)^2}{(1 + R_c^2)} r^{3/2}
\]
\[
0 \leq r \leq R_c
\]
\[
u = \left(\frac{R_c}{4 \mu} \frac{dP}{dz}\right)((R^2 - R_c^2) - \frac{8 (R_c^{1/2} - R_c^{3/2})((R - R_c^{1/2})^2 + H^2)((R - R_c^{3/2})^4 + H^2)}{(1 + R_c^2)^2}
\]
\[
+ \frac{(15(R - R_c) + H^2)(8(R - R_c) + H^2)(R - R_c)^2}{(1 + R_c^2)^2}
\]
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\]
\[
+ \frac{(R - R_c)^2}{(1 + R_c^2)} \right)^{1/2}
\]
\[
+ \frac{(4 R_c^4 + H^2)(R - R_c)^2}{(1 + R_c^2)} r^4
+ \frac{(8 H^2)(R - R_c)^2}{(1 + R_c^2)} r^4
\]
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\]
\[
0 \leq r \leq R_c
\]
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+ \frac{(5 R_c + H^2)(R - R_c)^2}{(1 + R_c^2)} \right)^{1/2}
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+ \frac{(8 H^2)(R - R_c)^2}{(1 + R_c^2)} r^4
\]
\[
+ \frac{(5 R_c + H^2)(R - R_c)^2}{(1 + R_c^2)} \right)^{3/2}
\]
\[
0 \leq r \leq R_c
\]

The flow rate for the blood flow with transverse magnetic field is,
\[
Q = \frac{B}{4} \int_0^R 2 \pi u r \, dr
\]
By the help of equation (11) and equation (12), flow rate can,
\[ Q = \left( \frac{R_e \varepsilon}{2\mu} \right) \left( \frac{2\partial P}{\partial z} \right)^{2/3} \left( \frac{(4\mu R_e)^{1/3}}{R^2} + \frac{8(R_e/R)^2 + H^2}{3(1 + R^2)} + \frac{(15R + H^2)(8(R_e/R)^2 + H^2)}{8R^2} + \frac{H^2(R_e/R)(2(R_e/R)^{2/3} + H^2)}{(1 + (R_e^2/R)^2)} + \frac{(8(R_e/R)^{2/3} + H^2)H^4}{(1 + (R_e^2/R)^2)} \right) \tag{14} \]

From equation (13) pressure gradient is written as follows,
\[ \frac{\partial P}{\partial z} = \left( \frac{R_e \varepsilon}{2\mu} \right) \left( \frac{(4\mu R_e)^{1/3}}{R^2} + \frac{8(R_e/R)^2 + H^2}{3(1 + R^2)} + \frac{(15R + H^2)(8(R_e/R)^2 + H^2)}{8R^2} + \frac{H^2(R_e/R)(2(R_e/R)^{2/3} + H^2)}{(1 + (R_e^2/R)^2)} + \frac{(8(R_e/R)^{2/3} + H^2)H^4}{(1 + (R_e^2/R)^2)} \right) \tag{15} \]

The dimensionless expression for resistance to flow, using
\[ \lambda = \left( \frac{P_i - P_0}{Q} \right), \tag{16} \]

Using the boundary conditions, \( P_i \) is pressure at \( z = 0 \) and \( P_0 \) is the pressure at \( z = L \). The resistance to flow can be written as,
\[ \lambda = \left( \frac{HR_e e}{np} \right)^{2/3} \left( \frac{L_0}{4\pi R_0} \right)^{7\delta^2} A_i \left( \frac{L_0}{8\pi R_0} \right)^{7\delta^2} \left( \frac{L_0}{2} \right) \]

\[ + \frac{L_0}{2\pi R_0} A_i \left( \frac{20\delta^2(L_0d)^{m+1}}{4R_0^2} - \frac{5\delta^2}{2R_0} \right) \left( \frac{L_0}{8\pi R_0} \right)^{7\delta^2} \left( \frac{L_0}{2} \right) + \frac{L_0}{2\pi R_0} \left( \frac{3\delta^2}{2R_0} A_i \left( \frac{5\delta^2}{2R_0} \right)^{2/3} \right) \]

\[ + \frac{3\delta^2(L_0d)^{m+1}}{2R_0^2} + \frac{6\delta^2(L_0d)^{m+1}}{2R_0^2} + \frac{L_0(L_0d)^{m+1}}{4\pi R_0^2} \left( \frac{L_0}{2} \right)^{7\delta^2} + \frac{L_0}{2\pi R_0} \left( \frac{3\delta^2}{2R_0} A_i \left( \frac{5\delta^2}{2R_0} \right) \right) \]

\[ + \frac{6\delta^2(L_0d)^{m+1}}{4R_0^2} + A_i \left( \frac{20\delta^2}{4R_0^2} \left( \frac{5\delta^2}{2R_0} \right)^{2/3} \right) \left( \frac{L_0}{2} \right)^{2/3} \]

\[ + \left( \frac{(L_0d)^{m+1}}{2\pi R_0} \right)^{2/3} \left( \frac{L_0}{2} \right)^{2/3} + \frac{L_0}{2\pi R_0} \left( \frac{3\delta^2}{2R_0} A_i \left( \frac{5\delta^2}{2R_0} \right)^{2/3} \right) \]

\[ + \left( \frac{(L_0d)^{m+1}}{3R_0^2} \right)^{3/2} \left( \frac{L_0}{2} \right)^{3/2} \]

\[ \frac{\delta^2}{2\pi R_0} \left( \frac{5\delta^2}{2R_0} \right)^{2/3} \left( \frac{L_0}{2} \right)^{2/3} + \frac{L_0}{2\pi R_0} \left( \frac{3\delta^2}{2R_0} A_i \left( \frac{5\delta^2}{2R_0} \right) \right) \]

\[ + \left( \frac{(L_0d)^{m+1}}{3R_0^2} \right)^{3/2} \left( \frac{L_0}{2} \right)^{3/2} \]

\[ \text{where } A_i = R_e e(R^2 + (8 + H^2)R^4) \left( \frac{1}{1 + R^2} \right)^{2/3} + \frac{(8 + H^2)(15 + H^2)R^6}{(1 + R^2)^{2/3}} + \frac{8 + H^2}{(1 + R^2)^{2/3}} \]

The shearing stress at the wall can be written as,
In order to have an estimate of the quantitative effects of various parameters such as Hartmann number (H= 0, 0.2, 0.6, 1), red cell (M = 0, 2, 4), stenosis shape parameter (m = 2...11) involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, velocity profile and wall shear stress for diseased system associated with stenosis due to the local deposition of lipids have been determine. Effects of magnetic field on flow characteristics in presence of stenosis have been depicted in figure 2 to 8. Fig. 2 depicts the variation of axial velocity with radial co-ordinate. It is clear from the figure that of magnetic field reduces the velocity of blood. The development of stenosis accelerates the velocity of plasma decreasing. In a healthy artery, the wall shear stress is approximately 15 dy/cm². The interior surface will be prone to high blood pressure. Therefore the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma. These results are consistent with the observation of [15]. Fig.5 shows the results for resistance to flow with axial distance for different values of α. Resistance to flow decreases for increasing value of axial distance. It is also seen that the resistance to flow decreases as α decreases.

To capture the results for wall shear stress in the presence of externally applied magnetic field, the graphs have been plotted in figure 6 to 8. The fig.6 shows the results for wall shear stress (τ) for different values of stenosis size (δ/Ro). Wall shear stress (τ) increases as stenosis size increases and decreases as axial distance increases. It is concluding from these results that the wall shear stress decreases as the stenosis grows and remains constant outside from the stenotic region. It is also seen that wall shear stress is decreases when an external magnetic field is applied. This result is qualitative agreement with the observation of [8, 11]. It is observed that reduction in velocity, resistances to flow and wall shear stress are more pronounced in central region of the pipe and results are compared with [10]. Fig.7 shows the results for wall shear stress for different values of Hartmann numbers. Wall shear stress (τ) decreases as Hartmann numbers increases and decreases as axial distance increases. Fig.8 shows the results for wall shear stress for different values of α. Wall shear stress (τ) decreases for increasing value of axial distance. It is also seen that the wall shear stress decreases as α decreases. In a healthy artery, the wall shear stress is approximately 15 dy/cm². The interior surface will be damaged once the wall shear stress reaches a value higher than 400 dyn/cm² [7]. Therefore to determine the critical flow condition, prediction of wall shear stress using numerical experiments become necessary. It is clear that of magnetic field reduces the velocity of blood, resistance to flow and wall shear stress. It is observed that reduction in velocity, resistances to flow and wall shear stress are more pronounced in central region of the pipe. Our results are similar to those obtained by [9]. Therefore magnetic field can be effectively utilized to deaccelerate the blood flow in flow problems. The effect of magnetic field on blood has been analyzed theoretically by treating blood as an electrically conducting fluid [18]. The conductive flow in the presence of a magnetic field induces voltages and currents resulting in decrease in the flow using 2.35T magnet, the flow of 15% Nacl was reduced by less then 1%. A detailed theoretical analysis including induced current through the

\[
\tau = (\mu \left( \frac{\partial P}{\partial Z} \right)^{3/2} \frac{R e \varepsilon}{R^2} \left( \frac{2 R H^2}{1 + R^2} + \frac{4 R^3 (5 + H^2) (8 + H^2) (15 + H^2)}{(1 + R^2)^{3/2} (2 + R^2)^{3/2}} \right)
\]

Results and Discussion

In order to have estimate of the quantitative effects of various parameters such as Hartmann number (H= 0, 0.2, 0.6, 1), red cell (M = 0, 2, 4), stenosis shape parameter (m = 2...11) involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, velocity profile and wall shear stress for diseased system associated with stenosis due to the local deposition of lipids have been determine. Effects of magnetic field on flow characteristics in presence of stenosis have been depicted in figure 2 to 8. Fig. 2 depicts the variation of axial velocity with radial co-ordinate. It is clear from the figure that of magnetic field reduces the velocity of blood. The development of stenosis accelerates the velocity of plasma decreasing. In a healthy artery, the wall shear stress is approximately 15 dy/cm². The interior surface will be prone to high blood pressure. Therefore the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma. These results are consistent with the observation of [15]. Fig.5 shows the results for resistance to flow with axial distance for different values of α. Resistance to flow decreases for increasing value of axial distance. It is also seen that the resistance to flow decreases as α decreases.

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\]
blood vessel wall and surrounding tissue resulted in small decrease in the rate of flow due to the magnetic field.

0.05% for 1T and 0.47% for 3T [19].

Fig.2. Variation of axial velocity with radial direction for different hartman number

Fig.3. Variation of resistance to flow with stenotic region for different magnetic number
Fig. 4. Variation of Resistance to flow with axial distance for different hartman number

Fig. 5. Variation of Resistance to flow with axial distance for different $\alpha$
Fig. 6. Variation of wall shear stress with axial distance for different thickness of stenosis

Fig. 7. Variation of wall shear stress with axial distance for different Hartman number
Concluding Remarks:

In this present study, velocity profile, resistance to flow and wall shear stress are obtained when blood is assuming as Bingham fluid model, electrically conducting fluid, so that the effect of magnetic field on blood flow through an artery can be observed. Looking at the importance of the hydrodynamic factors in the understanding of blood flow and atherosclerotic diseases, it may be said that the present model could be useful for investigating blood flow through stenosed artery, in particular in diseased stage when blood is no longer a Newtonian fluid; it has stress with magnetic effects. It is noticed that the resistance to blood flow and wall shear stress decreases for different magnetic fields. The magnetic field is to decrease the resistance to flow due to irregular boundaries. So the magnetic field can be effectively utilized to deaccelerate the blood flow in flow problems. The present study is able to predict the main characteristic of the physiological flows and may have played some important role in biomedical application.

References:


